Journal of Motor Behavior, 1992, Vol. 24, No. 1, 67-83

Dynamical Aspects of Learning an Interlimb Rhythmic Movement Pattern

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ABSTRACT. Learning a bimanual rhythmic task is explored from the perspective that motor skill acquisition involves the successive reparameterization of a dynamical control structure in the direction of increasing stability, where the intentional process of reparameterization is itself dynamical. Subjects learned to oscillate pendulums held in the right and left hands such that the right hand frequency was twice that of the left (2:1 frequency lock). Over 12 learning sessions of 20 trials each, we interpreted the decreasing fluctuations in the frequency locking to be an index of the increasing concavity of the underlying potential, a measure of stability; the time required to achieve the 2:1 pattern was interpreted as indexing the relaxation time of an intentional dynamic. Power spectral analyses of the phase velocity ratio exhibited two strategies for acquiring the interlimb movement pattern: (a) adding spectral peaks at integer multiples of the left hand frequency or (b) distributing power across many frequencies in a $1/f$-like manner. Results are discussed in terms of the promise of a dynamical approach to learning coordinated movements.

Key words: kinematic feedback, motor learning

When a person walks across the floor, the organization of the locomotory act is a consequence of a number of constraints. Cognitive constraints will establish the intentional elements of the act (for example, the direction and speed of the locomotory trajectory) on the basis of the goal to be achieved. Neural and metabolic constraints will coordinate the microstructural contractile machinery that is necessary for generating the forces and elastic potentials sufficient, in this case, to propel the pendular masses of the legs and arms. Further, these neural constraints must act in concert with physical constraints that are dictated by the macroscopic machinery of the movement system and its interface with the environment. In this case, the physical constraints comprise the pendular masses moving in a gravitational field, the inertial reactive forces that are caused by them, frictional forces associated with the metabolic machinery, and the macroscopically defined elastic potentials and forcing functions of the musculature.[1] Finally, perceptual information (i.e., haptic, visual) that apprises the person not only of the environmental properties that may affect the achievement of the goal at hand (e.g., rough terrain, obstacles, etc.) but also about the states of the limbs themselves, constrains the organization of an action and can be said to tune, thereby, the control processes for the speed, direction, and stability of the movement.

A number of investigators have chosen to concentrate on the macroscopic definition of the movement system and the dynamical properties of its machinery as the nexus of these constraints. The basic properties of movement systems from this dynamical perspective are as follows: On the basis of the cognitive constraints on the movement that are specified by the intentional state of the actor, synergies are formed between an actor's muscles, joints, and the force-producing mechanisms of the neural and metabolic machinery to produce autonomous control structures that are responsible for the execution of a movement. These functional subsystems have been variously named special-purpose devices (e.g., Fowler & Turvey, 1978), task-specific devices (e.g., Bingham, 1988), and coordinative structures (e.g., Easton, 1972; Turvey, 1977). The assembly process of these subsystems results in physical constraints that are given by the basic macroscopic form of the movement system (e.g., which degrees of freedom are linked, which are metabolically forced, which are passive, etc.) and the environmental

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forces (e.g., gravity) and objects (e.g., balls to be thrown, etc.) with which it coordinates. The functioning of a subsystem is claimed to be relatively autonomous because it is based upon strategies of self-organization that are found at many scales of nature (Kugler & Turvey, 1987; Turvey, 1990). That is, these macroscopically defined and physically constrained subsystems are characterized by the "free interplay of forces and mutual influences among the components tending toward equilibrium or steady states" (Kugler, Kelso, & Turvey, 1980, p. 6) and, hence, self-organize.

These dynamical control structures are also constrained informationally. Fowler and Turvey (1978) proposed that, for each degree of freedom of the coordinative structure left unconstrained by muscle linkages, there is a variable of the animal-environment system—specified by perceptual information—that constrains its magnitude. These variables may include distal segments of the actor's own body or of another actor's body (Schmidt, 1990; Schmidt, Carello, & Turvey, 1990) that are coordinated by virtue of haptic information or optical information, respectively. Because of these informational linkages, the functional form of the control structure gains closure. That is, the extent of the control structure spans both the actor and his or her environment. Hence, its basic unit of functioning is an event (Fowler & Turvey, 1978) or a perceiving-acting cycle (Bootsma & Van Wieringen, 1990; Shaw & Kinsella-Shaw, 1988). Accordingly, an actor actually controls the functional description of the perceiving-acting cycle or event, when he or she produces a movement, rather than just a functional description of his or her own body.[2]

Dynamical Approach to Learning

Acquiring a new movement pattern, from the dynamical perspective on coordination of movements, is tantamount to learning the laws that govern a dynamical control structure (Shaw & Alley, 1985). When this occurs, the optimal organization of the components of the control structure can be formed. Fowler and Turvey (1978) suggested that learning the laws means that an actor has to become perceptually attuned to the consequences of different configurations or scalings of the components of a control structure: "A [movement pattern] has structure, and discovering an optimal self-organization is in reference to those variables of stimulation corresponding to environmental and biokinematic relations that specify the essential features of the [movement pattern] the actor is to perform" (p. 6). Fowler and Turvey (1978) further suggested that the optimal organization arises when an actor has configured his or her action system so that the following two principles are implemented: (a) The reactive forces of the limbs and their interaction with the environment are used so that they can be "largely responsible" for producing the desired trajectory (Bernstein, 1967, p. 19) and (b) additional degrees of freedom are used, thereby increasing the number of controllable parameters within the control structure so that the movements may be performed more fluidly (Bernstein, 1967; Newell & Van Emmerik, 1989; Van Emmerik & Newell, 1990). Further, it has been suggested that early in the learning process the learner's problem is to set up the necessary relationships among the control structure's components. Newell (1985) described the early stages of learning as the assembling of the appropriate relative motions or topological relations among action system components. Later in the learning process, the learner's problem is to achieve the correct magnitudes of these relative motions (scaling them quantitatively) so that the goal can be accomplished more fluidly.

But how is an actor guided to these configurations by variables of stimulation that allow for the use of additional degrees of freedom and reactive forces? It has been suggested that to acquire a new movement pattern an actor perceptually explores the dynamical workspace of the control structure (Fowler & Turvey, 1978; Kugler & Turvey, 1987; Newell, Kugler, Van Emmerik, & McDonald, 1989). Each parameterization of the dynamical control structure that is assembled to initiate a new movement pattern is characterized by an attracting region of a potential field (see below) that has a stability specific to the parameterization (Kugler & Turvey, 1987). The initial parameterizations of the control structure assembled to execute a movement pattern just being learned have equilibrium regions that are not very stable or that keep collapsing and need to be constantly reassembled. Other parameterizations on subsequent occasions (e.g., cycles, trials, sessions) will be more or less stable. How the stability of the equilibrium regions changes as the parameters of the control structure are manipulated from occasion to occasion will specify how these parameters have to be manipulated to move toward the optimal equilibrium state. The optimal equilibrium configuration of the control structure is approached in time because at each parameterization the magnitude and direction that each parameter needs to be changed is specified by the change in stability since the last parameterization.

Dynamical Systems and Stability

The present article is an attempt to further the ideas of the dynamical basis of skill acquisition by providing a methodology that allows an estimation of the stability of control structures underlying the learning. To do this, we first discuss the concept of stability as it occurs in dynamics.

Any physical system can be represented by the possible states into which it can enter (Abraham & Shaw, 1982). If one chooses relevant variables to quantify a system's states, the relationships among these variables can be used to create a geometric model for the set of all idealized states. Such a model is called the *state space* of the system. The history of a system's states in a state-space model is called a *trajectory*. How a dynamical system will evolve over time can be represented by finding the rates of change of trajectories at each point in the state space. Then the values are represented as vectors (the magnitude of the rate of change in a particular direction). The set of all vectors at all points in a state space is called the *vector field* representation of the dynamical system. A dynamical system has special trajectories that are used to categorize the kind of dynamic the system displays and represents the asymptotic behavior of the system as time goes toward infinity. These portions of the state space are called *limit sets*, and the most probable ones (i.e., the ones most frequently observed empirically) are called *attractors*. The simplest is when all trajectories in a portion of the state space are drawn to a single point. This equilibrium point is called a *point attractor*. The rate of change of a trajectory at this point is zero. Also, the trajectories could be attracted to a closed orbit within the state space. This *periodic* or *limit cycle attractor* represents some oscillation of the states of the system that has a period or frequency of oscillation associated with it. Other higher-order attractors exist (quasiperiodic attractors, chaotic attractors) that have limit sets of more complex topologies.

Often a dynamical system is represented by a state space plus a variable that represents a potential that governs the dynamics or time evolution of the system. The potential (in principle) represents the conserved quantity in state space that underlies the flow (i.e., the way that the trajectories evolve) of the system. In the systems of classical mechanics, the potential is usually represented by total or potential energy. Recently, there has been speculation as to whether or not new conserved quantities are necessary for describing the dynamics of perceiving-acting systems (Shaw & Kinsella-Shaw, 1988). Regardless of the specific identity of the potential, differences in the magnitude of the potential over the state space will cause trajectories in the state space to flow to those regions where the potential is less. The rates of change of state-space properties (the vector field) will be governed by the rate of change in potential (forces) in the different parts of the space. Those regions of the state space where the potential is least are the regions of attraction and define the attractor of the system. In what follows, we speak of these areas as the bottoms of wells in the field set up by the potential.[3]

The degree of attraction or the stability of an attractor is determined by the degree of concavity of the sides of the potential well. If one knows the potential function, $V(\omega)$, that describes the potential field of an attractor over a single state variable ω then $d^2V/d\omega^2$ measures the degree of concavity of the attractor's potential well. Potential wells for which $d^2V/d\omega^2 > 0$ are stable. Further, the degree of attraction or stability can be measured empirically by the relaxation time of the system, namely, the time, τ_{rel}, a trajectory takes to return to the equilibrium position from a perturbation of a given force F_p that takes it away from that position. A linear approximation of the τ_{rel} is the magnitude of $d^2V/d\omega^2$ (Schöner, 1989). Alternatively, if the relaxation time of the oscillator is known, the concavity of the well can be estimated. Figure 1 displays three potential fields $V_1(\omega)$, $V_2(\omega)$, and $V_3(\omega)$ such that the $d^2V_1/d\omega^2 > d^2V_2/d\omega^2 > d^2V_3/d\omega^2$. If a perturbation force F_p is applied to the three fields, the states of the three systems become ω_1, ω_2, and ω_3, respectively. Because of the decreasing concavity of the wells, the time to return to the bottom of the well, τ_{rel}, is greater for V_3 than V_2 and greater for V_2 than V_1.

In systems that are subject to stochastic fluctuation (such as dissipative structures in which macroscopic dynamical behavior is formed out of the dissipation at a microscale containing many degrees of freedom), the stationary behavior of the system at equilibrium is not static but is circumscribed to a region of the potential well. A number of facts are known about such systems. The size of this region is determined by (a) the magnitude of the stochastic forces Q_r and (b) the concavity ($d^2V/d\omega^2$) of the potential function. The observed magnitude of the fluctuations (e.g., distribution of observed states or its standard deviation) in variable ω will be a function of the size of this region. If one can assume that the stationary probability distribution of the system in question is gaussian

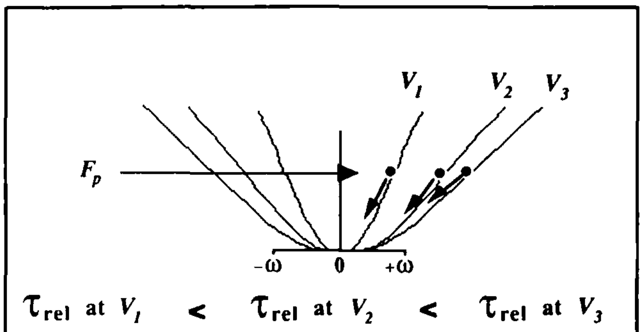

FIGURE 1. Wells in the potential fields $V_1(\omega)$, $V_2(\omega)$, and $V_3(\omega)$, showing how relaxation time, τ_{rel}, increases with decreasing concavity, under constant perturbation force, F_p.

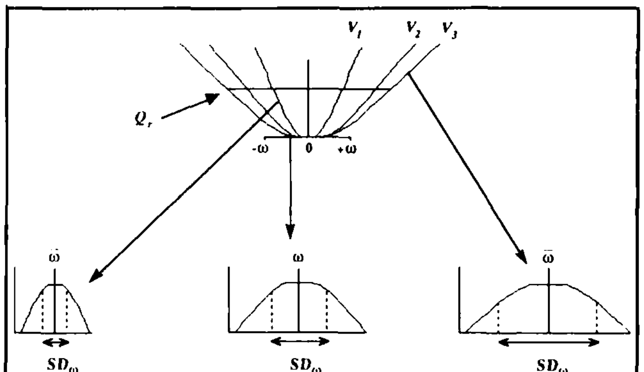

FIGURE 2. Potential wells and respective fluctuations as indexed by standard deviation of ω, under circumstances of constant stochastic force, Q_r.

(for simplicity), then the relaxation time can be determined from the standard deviation as follows: $\tau_{rel} = 2SD^2/Q$ (Schöner, 1989). If one assumes that the magnitude of the random noise in the system is constant ($Q_r = c$), then $d^2V/d\omega^2$ completely determines the relative magnitude of the observed fluctuations (Figure 2).

The question arises: How are these physical concepts to be applied to intentional, biological systems? That is, are causal, physical descriptions appropriate for systems that are ultimately governed by final causes or goals? The assumption here, following Kugler and Turvey (1987), is that, although intentional agents are complex physical systems that have on-board stores of chemical fuel that allow behaviors (such as walking up hills rather than rolling down them) that on short time scales appear noncausal, they nonetheless are ultimately subordinate to all of the causal laws that affect physical systems. Hence, intentional agents as a consequence of being in a goal state can assemble an action control structure that is governed by causal, physical dynamics. This control structure can coordinate the degrees of freedom of an action system in a self-organizing fashion. Instead of separating biological systems from physical regularities, intentions from this point of view are extraordinary boundary conditions on natural laws (Kugler & Turvey, 1987).[4]

Learning and Stability

If a coordination of movements is governed by a dynamical control structure, then, under some alternative, macroscopic description, a function can be used to describe its evolution of states, namely,

$$F(x_i, t, c_i, \xi) = 0,$$

where x_i are state space properties, t is time, c_i are control parameters that are manipulated to change the topology (and, hence, often the stability of the system's dynamic, and ξ is a stochastic noise process (Jackson, 1989; Schöner, Haken, Kelso, 1986). From the dynamical perspective, coordination can be defined as the setting up of this function, control as the assigning of values to its variables, and skill as the implementing of an optimal organization of these variables that reduces its instability to a minimum (Kugler et al., 1980; Newell, 1985).

The functioning of a dynamical control structure and its optimization over repeated performances are two separate processes. As stated above, the optimization process is taken to be an intentional, goal-directed activity. Some researchers have attempted to model this learning process itself dynamically (Schöner, 1989; Shaw & Alley, 1985). If, through an exploration of a control structure's perceptual workspace, the learner follows the gradient that specifies the greatest increase in stability (Fowler & Turvey, 1978; Kugler & Turvey, 1987; Newell et al., 1989), then this trajectory (which takes place at a longer time scale than the action itself) can be considered the product of a dynamical system. What needs to be hypothesized is a higher-order intentional control structure that governs the optimization of the lower-order action system control structure. The functioning of this higher-order structure is also dynamical—that is, governed by the "free interplay of forces and mutual influences among components tending toward equilibrium or steady states" (Kugler et al., 1980, p. 6). This line of thinking has led Shaw and Alley (1985; see also Shaw, Kadar, Sim, & Repperger, 1992) to propose that the law form of hereditary mechanics, characterized as a "historical series over which initial conditions are 'updated' to reflect hereditary influences" (p. 295), is necessary to capture the phenomena revealed in learning curves. Hereditary equations take an integro-differential form such that the hereditary function (the higher-order control structure) sets up the initial conditions of the other function (lower-order action system control structure) and hence, reparameterizes it for the next occasion (cycle, trial, or session). The hereditary function's relaxation to an equilibrium state would correspond to the optimal parameterization of the lower-order function. The crucial aspect of this way of theorizing is that an intentional process (i.e., the moving toward a targeted movement pattern) is being imbued with dynamics: The attainment of a goal state is achieved by the setting up of a higher-order control structure that has an equilibrium state at the goal state.

A similar scheme was used by Schöner (1989) and Zanone and Kelso (in press) in their modeling and empirical investigation (respectively) of the time evolution of the dynamics underlying the production of a specific movement pattern, namely, the relative phasing of 1:1 frequency-locked finger movements at 90°. The preferred coordinative states of bimanual phasing occurred at relative phase relations of 0° and 180°. These two regimes represent the intrinsic dynamics of the movement control structure and as such form the initial conditions for the learning of any new patterning of relative phase. To model the behavior of the movement system during the learning of a new relative phase pattern, Schöner (1989) endowed both environmental and memorized information with dynamics that have equilibrium states at the to-be-learned pattern. The combined system of these intentional dynamics and the intrinsic dynamics underlying the action control structure provides a model of the time evolution of the system from its producing an intrinsic relative phase pattern (0° or 180°) to its producing a learned relative phase pattern (e.g., at 90°).

Further, Schöner (1989) identified the time scale of the learning process, τ_{learn}, as an estimation of the relaxation time of the intentional dynamics controlling the learning (Figure 3). τ_{learn} can be a measure of the stability or strength of the control structure assembled to achieve the goal of learning the new movement pattern. Further, he suggested that the shape of the learning curve reflects the changing topology of the underlying attractor layout of the combined system of the intentional dynamics and the action control structure dynamics. Hence, a dynamical model, such as the one he used to represent the learn-

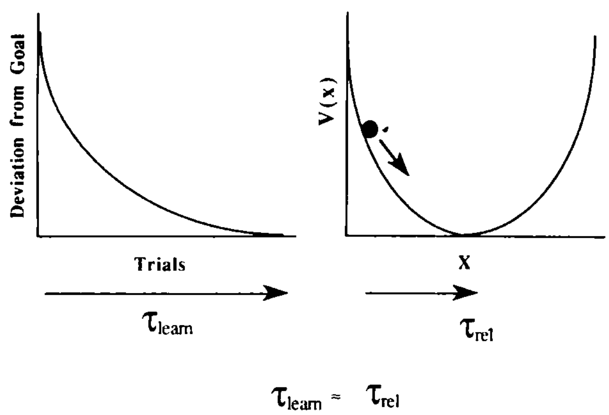

FIGURE 3. Proposed relationship between a prototypical learning curve and its associated time, τ_{learn}, and an intentional dynamic and its associated time of stabilization or the relaxation time, τ_{rel}.

ing of a new 1:1 frequency-locked phase pattern, can predict and explain the appearance of discontinuous jumps in learning curves. Such bifurcations would represent nonlinear changes on the potential surface that represents the attractor layout (Schöner, 1989). Zanone and Kelso (in press) provided empirical support for these ideas by showing the evolution of the attractive regions of relative phase over the course of practicing a new relative phase pattern at a 90° phase difference.

Experiment Preview

In the present article, we apply the above ideas to the acquiring of an interlimb coordination in which two hand-held rods with masses attached at their lower ends are moved simultaneously in pendular fashion by motions at the wrists, with the right pendular motion occurring at twice the frequency of the left pendular motion. An important feature of investigating the learning in the "wrist-pendulum" paradigm (Kugler & Turvey, 1987) is that the rhythmic units have well-defined frequency preferences as gravitational pendulums. From the experimenter's perspective, this feature means that there are benchmark values to which observed frequencies can be compared; it also means that the experimenter has a convenient way of manipulating competing frequencies in bimanual rhythmic coordination. From the subject's perspective, it means that efforts to impose a particular frequency pattern on the rhythmic units may be aided or opposed by the natural tendencies of the units, depending on the match between the ratio of their uncoupled frequencies and the ratio of coupled frequencies that is to be learned. In the experiment reported in the present article, the eigenfrequencies of the component rhythmic units were identical. Thus, the challenge was to acquire a 2:1 frequency-locking pattern with units whose ratio of uncoupled frequencies was 1:1.

Method

Subjects

Two male graduate students and 2 male faculty members at the University of Connecticut—the authors of this paper—acted as subjects. All subjects were right-handed. Three of the subjects acted as experimenters as well, instructing other subjects and acquiring data from them.

Materials and Apparatus

The hand-held pendulums consisted of an aluminum rod of 1 cm diameter attached to a wooden hand grip of 2.5 cm diameter and length 12 cm. The pendulums were 46 cm long, including the wooden grip and the rod. A cylindrical 0.2-kg weight was fastened to the end of the rod by means of screws built into the weight.

Subjects sat in a specially designed chair with arm rests to support their forearms. The arm rests were designed to restrict the subjects' swinging movements to the wrist or hand; the forearm was to be kept continually resting on the arm support. The chair also raised the subjects' legs with leg supports so that the legs did not interfere with the ultrasonic acquisition of the data.

Wrist-pendulum movement trajectories were collected using an Ultrasonic 3-Space Digitizer (SAC Corporation, Westport, CT). An ultrasound emitter was affixed to the end of each pendulum. An ultrasound "spark" was issued from each emitter 90 times per second. The digitizer registered each emission by using three microphones arranged to form a square grid. The digitizer calculates the distance of each emitter from each microphone, thereby pinpointing the position of the emitters in three-dimensional space at the time of the emission. This slant range information was stored for later use on a 80286-based microcomputer, using MASS digitizer software (Engineering Solutions, Columbus, Ohio). This software and analogous routines written on the Macintosh II use the slant range time series to calculate the primary angle of excursion of the pendulums and their phase angle velocities.

Procedure

Each subject performed 12 experimental sessions over the course of 3 months. The sessions were 1.5 weeks apart on average. Each session consisted of 20 trials, each trial lasting 32 s.

The subjects were given instructions in the first session. Their task was to swing a pair of identical hand-held pendulums at their wrists in a 2:1 frequency relation, with the right pendulum completing two cycles for every one cycle of the left pendulum. They were told to place their forearms squarely on the arm rests, to gaze straight ahead without looking at the pendulums, and to swing the pendulums smoothly back and forth. They were instructed to hold the pendulums tightly in their hands, so that as much of the rotation of a pendulum as possible was going on about the wrist joint rather than in the hand. Finally, they were instructed to oscillate the right pendulum twice the frequency of the left, in the most comfortable manner possible.

At the start of each trial, the subjects were allowed as much time as they needed to achieve what they felt was a comfortable 2:1 pattern. Data recording began after the subject indicated that he was ready. Subjects were allowed to rest between the trials of a session when they felt the need. No feedback was given on how well the goal of the task had been achieved during a session. The mean frequency-locking data for the previous sessions was sometimes reviewed by the subjects (because they were also experimenters) between sessions, however.

Data Reduction

The digitized displacement time series of the wrist-pendulum systems were smoothed using a triangular moving-average procedure with a window size of seven samples. Each trial was subjected to software analyses that determined the frequency of oscillation of each wrist-pendulum system, the ratio of these frequencies, the frequency ratio standard deviation, the time series of the ratio between the phase velocities of the two wrist-pendulum systems, the power spectra of this phase velocity ratio time series, and the total power associated with each of these spectra.

A peak-picking algorithm was employed to determine the time of maximum forward extension of the wrist-pendulum trajectories. From the peak extension times, the frequency of oscillation for the nth cycle was calculated as

$$f_n = 1/[\text{time of peak extension}_{(n+1)} - \text{time of peak extension}_n].$$

The mean frequency of oscillation for a trial was calculated from these cycle frequencies. The ratio of the frequency of each left hand cycle and the frequency of the right hand cycle immediately before it was calculated to produce the frequency ratio of each cycle. The mean frequency ratio for a trial was calculated from these cycle frequency ratios. The standard deviation associated with this mean was used as an index of frequency-locking stability for each trial. The mean frequency, mean frequency ratio, and mean frequency ratio standard deviation for a session were then calculated from the trial measures of a session.

The phase angle of each wrist-pendulum system was calculated for each sample (90/s) of the displacement time series to produce a time series of the relative phase angle. The phase angles at sample i (ϕ_i) were calculated as

$$\phi_i = \arctan(\dot{x}_i{}^*/\Delta x_i),$$

where $\dot{x}_i{}^*$ is the velocity of the time series at sample i divided by the mean angular frequency for the trial, and Δx_i is the displacement of the time series at sample i minus the average displacement for the trial.[5] The rate of change of the phase angle was calculated for each hand, and the ratio between these rates of change (Δleft ϕ_i/Δright ϕ_i) was calculated. This phase velocity ratio (which will be referred to hereinafter as the PVR) is used here because for 2:1 frequency-locking behavior it will yield a constant value of 2. It is not to be confused with the relative phase difference ($\phi_{1i} - \phi_{2i}$) that is used to evaluate 1:1 frequency locking. Both are indices of the relative motion or relative phasing of the two limbs, and the choice of one or the other should depend upon the nature of the task. The PVR time series would allow an evaluation of how the subject satisfied this task demand. To determine the magnitude and patterning of the variability associated with this time series, we performed a power spectral analysis on the PVR time series. The data first were windowed using a Parzens filter to reduce spectral leakage, and all of the trial spectra from a given session were averaged to reduce the error of the spectral estimate (Press, Flannery, Teukolsky, & Vetterling, 1988). Second, the total power of PVR was calculated for each session by summing the power at each frequency of the averaged spectra, excluding the DC component at the zero frequency. This measure was used as a summary of the variability of the relative motion between the left and right wrist-pendulum systems.

Results and Discussion

The different subjects found the task in the initial session difficult to varying degrees. All were surprised that the 2:1 coordination with the wrist-pendulum system was as difficult as it was compared to other common 2:1 coordinations (e.g., finger tapping). The difficulty of the task could be a consequence of the fact that a pendulum in the gravitational field has a dynamics of its own or result from the lack of discrete indicators about the relative motion of the units (for example, when a surface is contacted by a finger completing its cycle, as in tapping). The confidence of the subjects grew throughout the sessions, and by about Session 8 all could accomplish the task quite easily.

Frequency of Oscillation

The mean frequencies of oscillation produced by the 4 subjects across the 12 sessions are presented in Figure 4 for the left hand (top) and the right hand (bottom). Also represented in these figures are the frequency of the corresponding gravity pendulum with small amplitude oscillations (i.e., $< 30°$) (top) and twice this frequency (bottom). Observe that for Subjects 1, 2, and 4, frequencies were approximately constant for the 12 sessions. The frequencies of oscillation produced were, for the most part, slightly higher than that of a gravitational pendulum for the left hand and were, for the most part, slightly higher than that of twice a gravitational pendulum for the right hand. The left hands' data replicate past results that have modeled the comfort mode frequency of a hand-pendulum as a nondissipative pendulum in a gravitational field with a linear spring that represents the contribution to the frequency of the neuromuscular elastic potential (Bingham, Schmidt, Turvey, & Rosenblum, 1991; Schmidt & Turvey, 1989; Turvey, Schmidt, Rosenblum, & Kugler, 1988).

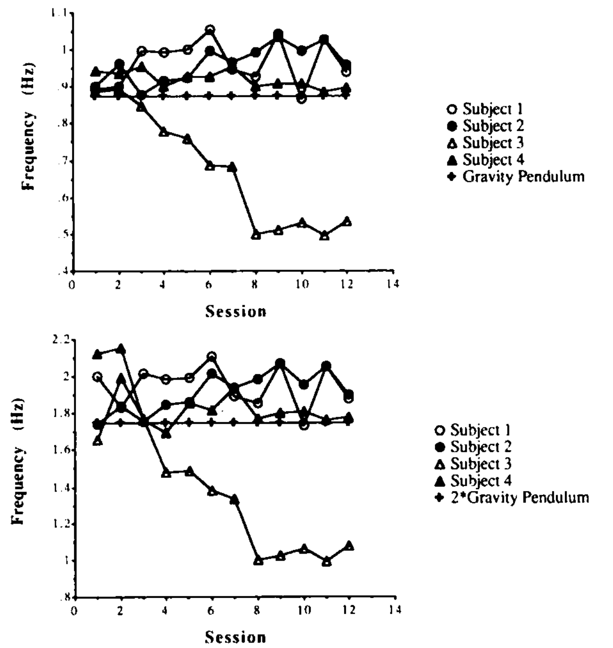

FIGURE 4. Mean frequency of oscillation for all subjects across all sessions for left (top) and right (bottom) hand. The frequency of oscillation of a corresponding gravity pendulum is shown with the data of the left hand, and twice the frequency of oscillation of a corresponding gravity pendulum with the data of the right hand.

These 3 subjects chose to swing the left hand wrist-pendulum at its comfort mode frequency and the right hand wrist-pendulum at twice this frequency. Subject 3 was different from the others, however, because he exhibited a nonstationary frequency of oscillation. In the first two sessions, this subject's left hand frequency approximated the gravitational frequency and the right hand frequency approximated twice the gravitational frequency. But thereafter, the frequencies steadily declined in tandem until in the eighth session the left hand reached one half the gravitational pendulum frequency and the right hand reached the gravitational pendulum frequency. The frequencies stayed at these magnitudes until the end of the experiment. Overall, all 4 subjects found comfort mode frequencies of oscillation near the gravitational pendulum frequency for one of the hands, and its doubling or halving in the other hand. Because the task requirements put no constraints on the frequencies of oscillation to be performed other than that the two hands remain 2:1 frequency locked and the right hand perform at a frequency twice that of the left hand, Subject 3's nonstationary frequencies did not violate the task requirements. It remains to be seen if this subject's frequency-locking behavior was affected by his choice in frequencies of oscillation.

Frequency Locking

The mean frequency ratio produced by each subject on the 240 trials of the experiment are presented in Figure 5. There was an obvious decrease in the deviation of ratios from 2:1 over sessions for all 4 subjects. The magnitude of these deviations and their rate of change across sessions, however, seemed to differ across subjects. These differences can be seen more easily in the mean absolute deviation from a 2:1 frequency lock for each session (Figure 6). In Session 1, the mean deviations from a perfect 2:1 frequency lock were .247, .064, .121, and .254 for

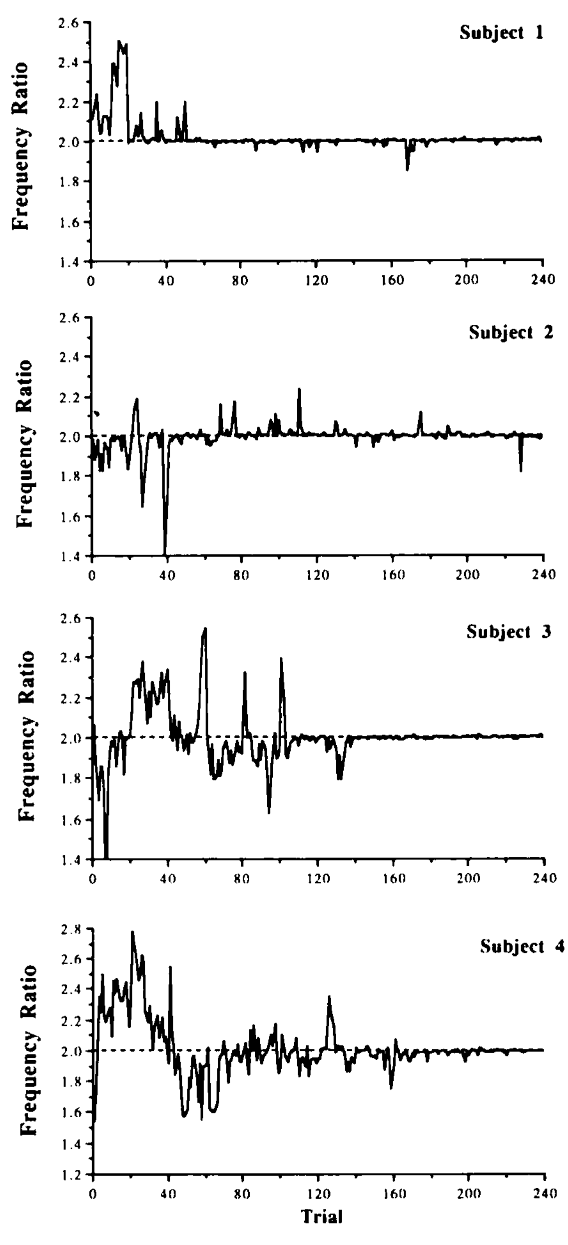

FIGURE 5. Mean frequency ratio across the 240 trials of the entire experiment.

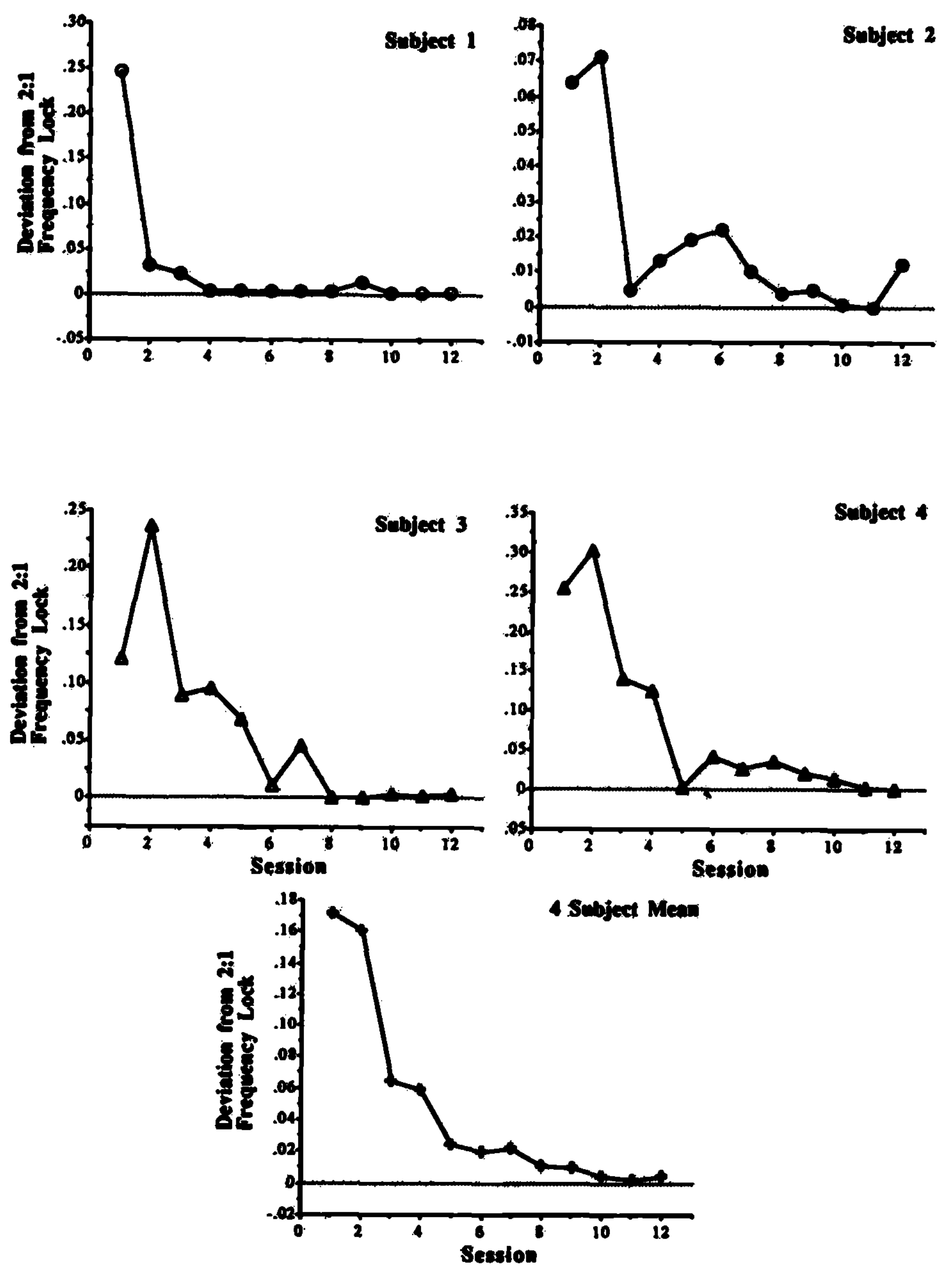

FIGURE 6. Mean absolute deviation from 2:1 frequency lock across the 12 sessions. Note the change in scale of the ordinate across subjects.

Subjects 1–4, respectively. In Session 12, mean deviations from a perfect 2:1 frequency lock were .002, .012, .003, and .001. If the left hand frequency of the subjects is estimated at 1.0 Hz, these numbers can be read as deviation from perfect 2:1 phase locking in seconds. The accuracy in frequency locking across the entire experiment increased from mean magnitudes on the order of 250 ms to mean magnitudes on the order of 1 ms.

Further, each subject's data can be fit by a second-order polynomial model, demonstrating the statistical significance of these decreases: Subject 1, $y = .202 - .055x + .003x^2$, $r^2 = .60$; Subject 2, $y = .079 - .017x + .001x^2$, $r^2 = .70$; Subject 3, $y = .217 - .04x + .002x^2$, $r^2 = .75$; Subject 4, $y = .356 - .076x + .004x^2$, $r^2 = .86$. All the coefficients were significant at $p < .05$ except for the quadratic term of Subject 3. These functions represent how the deviation from 2:1 frequency locking changes with time (sessions). If an arbitrary criterion is set for when learning has occurred (e.g., a deviation of .02), the amount of time (in sessions) necessary for learning can be calculated from these functions, namely, 5.0, 4.9, 8.8, and 7.0 sessions for Subjects 1–4, respectively. Further, assuming that the change in deviation across the sessions is caused by the dynamic of an intentional control process governing the learning, then the shape of this learning curve would be determined by concavity of the potential well associated with this learning dynamic's attractor. The concavity of the deviation regression model should reflect the concavity of this potential well. The deviation models' concavities are estimated by their second derivatives (see above), namely, .006, .002, .004, and .008 for Subjects 1–4, respectively. These values are presumably estimates of the strength or stability of the intentional dynamics that underlie the learning process. The reason that the learning time and the strength of the attractors do not order the same way across the subjects is that the initial deviation from 2:1 (which corresponds to the F_p in Figure 1) is different for the different subjects. This "initial perturbation" magnitude (that is presumably a reflection of the subject's natural predilection to produce 2:1 frequency lock) and the concavity of the potential well combine to effect the amount of time necessary for the dynamic to reach its equilibrium point.

Frequency Locking Fluctuations

The fluctuations in frequency locking were calculated as the standard deviation associated with the mean frequency locking for a trial. The mean frequency-locking fluctuations for a session were then calculated as the mean of the 20-trial standard deviations of a given session. The results across the 12 sessions can be seen in Figure 7 for the 4 subjects and the mean of the 4 subjects. All subjects showed significant decreases in fluctuations across the 12 sessions. In Session 1, the 4 subjects showed fluctuations across a great range of magnitudes (Subject 1, .122; Subject 2, .086; Subject 3, .148; Subject 4, .251). By Session 12, the range of fluctuations was circumscribed to the range of .06 to .09 (Subject 1, .065; Subject 2, .063; Subject 3, .081; Subject 4, .090). The decreases across the 12 sessions can be modeled by second-order polynomials for 3 of the subjects (Subject 1, $y = .130 - .015x + .001x^2$, $r^2 = .90$; Subject 2, $y = .084 - .007x + .0004x^2$, $r^2 = .69$; Subject 4, $y = .255 - .033x + .002x^2$, $r^2 = .84$) and a linear model for the remaining subject (Subject 3, $y = -.105x + .105$, $r^2 = .70$). All the coefficients were significant at $p < .05$.

All subjects reached the goal of the experiment: the learning of a 2:1 frequency-locking movement pattern. At Session 12, all 4 subjects had a minimal mean deviation from a perfect 2:1 frequency locking, and only small fluctuation about this state. As a matter of fact, the magnitude of the fluctuations was in the range exhibited by subjects in a 1:1 frequency-locking task. Roseblum and Turvey's (1988) results showed that for identical left and right hand pendulums in a 1:1 frequency lock the coefficient of variation (*SD/M*) for period and amplitude is in the 3–6 range. This is the same range as the coefficient of variation for the final sessions of the present experiment. The trend of decreasing fluctuations can be interpreted as indicating an increasing optimization of the dynamical control structure underlying the 2:1 frequency-locking movement pattern. Using the logic described in Figure 2, a standard deviation measures the distribution of the system's state over the potential field due to stochastic noise. This noise is an inherent property of the dynamical control structure's microcomponents. Assuming that the magnitude of this noise is constant over the different sessions, the decreasing magnitudes of the standard deviations reflect the decreasing concavity of the movement control structure's potential field. Figure 8 represents the evolution of such a field across the 12 sessions of the experiment, using the mean standard deviation magnitudes of the 4 subjects to estimate concavity. The evolution mean 2:1 frequency-locking state is also displayed on this graph by the change in position on the ω-axis of the minima of the potential well.

What is most surprising is that the patterning of mean deviations and fluctuations across sessions is so similar for Subject 3 and for Subjects 1, 2, and 4, in spite of the nonstationarity of Subject 3's frequency of oscillation. Regressions of sessions on mean deviation from 2:1 frequency locking and sessions on standard deviation of frequency locking have significant quadratic components, however, for Subjects 1, 2, and 4 but not for Subject 3. These latter contrasts point to the use by Subject 3 of a different strategy. Further, the major jump in Subject 3's fluctuations (Figure 7) was between Sessions 7 and 8, where this subject's oscillations moved from between the gravity frequency ratios to gravity frequency ratios. It is interesting that the fluctuations in frequency locking decreased dramatically at the stage of learning when the frequency of the left hand reached one half of the gravitational frequency.

Relative Phasing

The preceding analyses demonstrate that learning occurred and that this learning is amenable to a dynamical characterization that entails (a) the search for an optimal parameterization of an action system control structure and (b) a lawful basis to the intentional search procedure. The question that remains, however, is what aspects of the control structure are manipulated to provide the optimization. Intrinsic to the definition of a movement pattern are the relative motions of the body segments involved (Newell, 1985). Intuitively, the present frequency-locking task requires specific relative motion or phasing of the two wrist-pendulum systems. Further, relative phase has been used by some investigators as an order parameter that defines the basic dynamical patterns that underlie rhythmic movements in biological systems (Haken, Kelso, & Bunz, 1985; Kelso, 1990; Turvey, Rosenblum, Schmidt, & Kugler, 1986). Hence, it is credible to assume that the relative phasing of the 2:1 frequency locking is what is being tuned by variables of stimulation during the intentional search procedure.

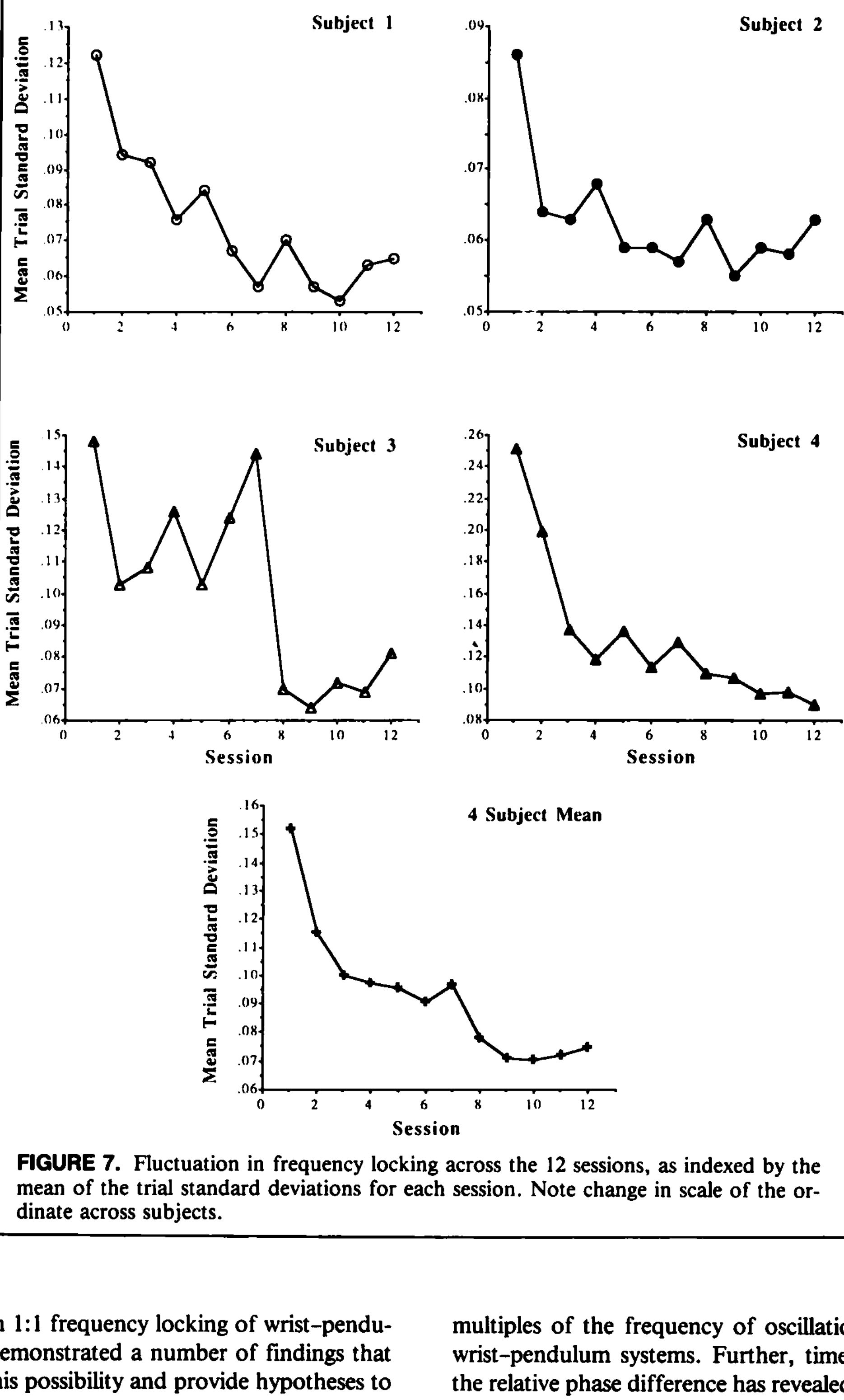

FIGURE 7. Fluctuation in frequency locking across the 12 sessions, as indexed by the mean of the trial standard deviations for each session. Note change in scale of the ordinate across subjects.

Past research on 1:1 frequency locking of wrist-pendulum systems has demonstrated a number of findings that give credence to this possibility and provide hypotheses to test it. First, past results have shown that the relative phase difference in 1:1 frequency locking tends toward two values, 0° and 180° (Turvey et al., 1986). These are commonly referred to as the symmetric and alternate phase modes, respectively. More recent research (Schmidt, Beek, Treffner, & Turvey, 1991) demonstrates that the relative phase difference tends not to be a constant value but often varies periodically at frequencies that are integer multiples of the frequency of oscillation of the coupled wrist-pendulum systems. Further, time series analysis of the relative phase difference has revealed that, as the coordination task becomes more difficult by making the wrist-pendulum systems more different in their masses and lengths and, therefore, eigenfrequencies (unlike the present experiment), (a) the total power of relative phase increases with task difficulty, (b) spectral peaks emerge at higher frequency multiples, and (c) the increased power is found at the spectral peaks (i.e., the spectral peaks grow in size).

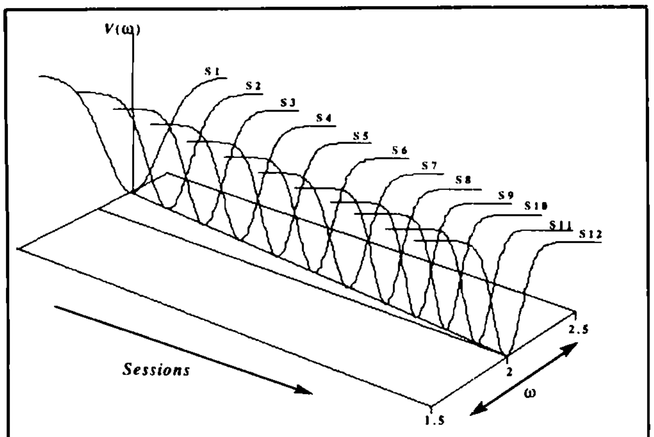

FIGURE 8. Schematic representation of the evolution of the movement control structure's potential field across sessions. Mean standard deviation magnitudes of all subjects are used to estimate concavity. Also depicted is the successive progression toward the goal of a 2:1 frequency lock (ω).

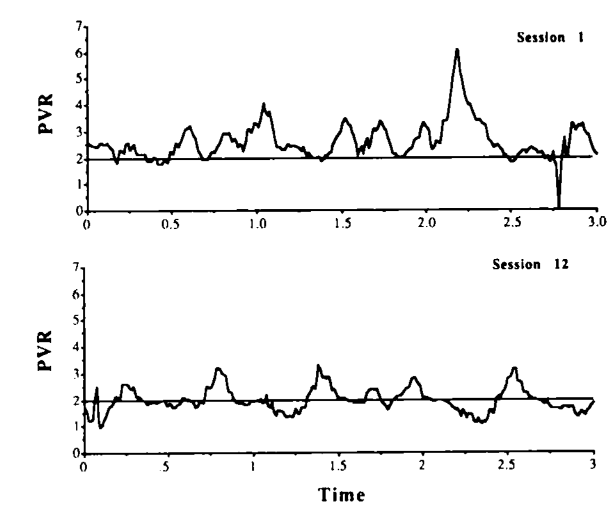

FIGURE 9. Examples of PVR time series from Sessions 1 and 12 for one of the subjects (Subject 4).

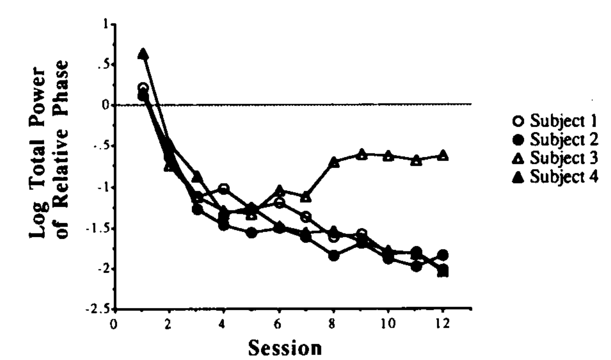

FIGURE 10. Log total power of the PVR across sessions for the 4 subjects. Note the divergence of Subject 3 after Session 7.

The increase of rhythmic processes in the relative phase (difference) spectrum has been hypothesized (Schmidt et al., 1991) to indicate changes in the manner in which the action control structure that is assembled is configured: As the task becomes more difficult, more rhythmic subcomponents emerge to deal with increasing reactive forces caused by the asymmetry of the wrist-pendulum systems' magnitudes. It has been proposed that these components may index an increasing number of discrete points within the cycle that are used as perceptual tuning points to keep the movement stable. In short, the claim is that as the coordination becomes increasingly difficult, the number of perceptual samples that are necessary increases. The increase of the number of these points suggests an increase in the amount of attention the performer must provide to the task at hand. Given these facts about the control of relative phasing in 1:1 frequency locking, one can hypothesize that the 2:1 frequency-locking control structure needs a certain number of relative phase subcomponents with perceptual tuning points in order for the movement pattern to be performed optimally. The relative phase spectrum during the process of learning the task should reflect the emergence of an optimal topology in spectra that corresponds to these rhythmic subcomponents.[6]

To measure the relative motion of the limbs in a 2:1 frequency locking, we used the phase velocity ratio (PVR) because it captures the spatial-temporal order in this kind of coordination: PVR should yield a constant magnitude of 2 for perfect 2:1 frequency locking. Two PVR time series are displayed in Figure 9 for Sessions 1 and 12 of a representative subject. As can be seen in the figure, the PVR was not constantly at 2, as one would expect from mechanical oscillators in a 2:1 frequency lock, but varied over the cycle in a periodic fashion. Further, the oscillatory nature of the PVR became more uniform as the sessions progressed. To estimate this change across the sessions, we used the total power of the PVR to obtain a measure of the changes in the overall activity of PVR. Figure 10 displays the change in log total power of the 4 subjects across the 12 sessions. All subjects show a sharp linear decrease in log total power for Sessions 1-3. (Only Subject 2's regression was significant, because of the small number of observations [i.e., 3].) Subjects 1, 2, and 4 demonstrate a further (less sharp) linear decrease for the rest of the sessions, whereas Subject 3 shows a slight but significant increase (all $p < .001$). Overall, these results point to a decrease in the activity of the relative motions of the wrist-pendulum systems, suggesting that in the later sessions the subjects were able to produce more stability (Figure 7) with less variability in the relative motion of the wrist-pendulum systems. The question remains whether or not this is because a particular relative phasing pattern was discovered that shaped the overall lesser amount of activity into just the form needed to harness the reactive forces encountered in the task. Further, Sub-

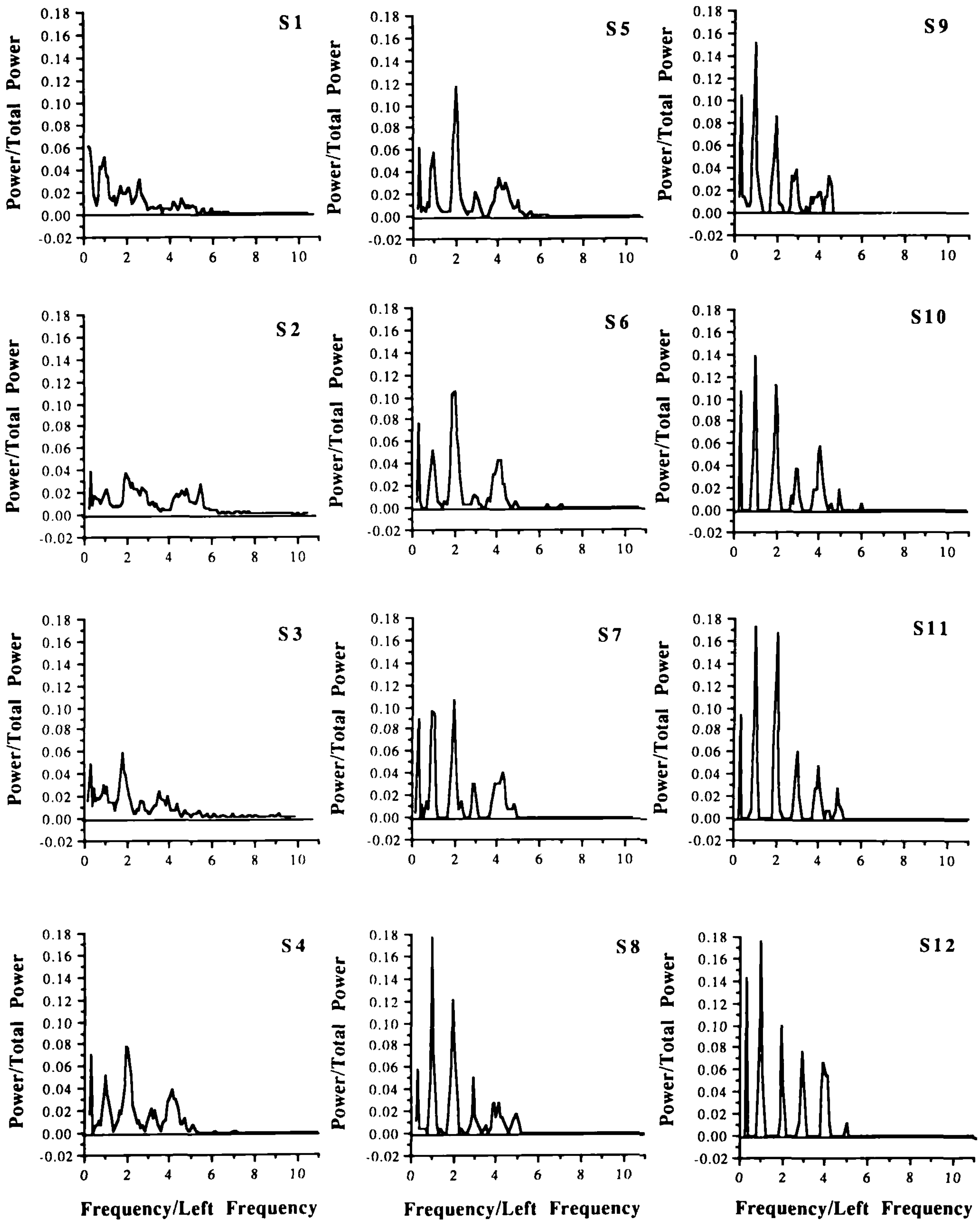

FIGURE 11. PVR power spectra across the 12 sessions for a single, representative subject (Subject 4), showing progressive delegation of power to integer multiples of the left hand frequency. Power at each frequency is normalized to the total power for that session. Frequency is normalized to the frequency of oscillation of the left hand.

ject 3 differed from the others in the amount of activity of relative phase required for the optimal performance of the 2:1 coordination. It remains to be seen whether or not this difference in magnitude of activity is coupled to a difference in the topology of the activity.

An examination of the changes in the shape of the PVR spectra across the 12 sessions suggests great uniformity in the spectra of Subjects 1, 2, 4 and the first 7 sessions of Subject 3. Figure 11 displays the spectra across the 12 sessions for a representative subject. Because the total power and frequencies of oscillation changed across the sessions, the power axis of the spectra was normalized by total power (power/total power) and the frequency axis was normalized by the frequency of oscillation (frequency/left hand frequency). The changes in these spectra across sessions are characterized by an emergence of peaks at integer multiples of the left hand frequency, an increase in the size of those peaks, and a decrease in power at frequencies in between the integer multiples. These changes can be summarized by plotting the percentage of the total power that is represented at these integer peaks across the sessions (Figure 12). Subjects 1, 2, and 4 demonstrated an increase in the percentage of total power at the peaks across the sessions ($p < .05$), whereas Subject 3 showed an initial increase (Sessions 1-3), a leveling off (Sessions 4-7) and a final decrease (Sessions 8-12). The total form of this subject's data was fit significantly by a second-order polynomial model (all coefficients $p < .05$).

The spectra of Subjects 1, 2, and 4, and those of the first 7 sessions of Subject 3, demonstrate the genesis of a relative phasing regime that solves the 2:1 frequency-locking problem. A possible interpretation of this solution is that a control structure is assembled with perceptual/rhythmic subcomponents at integer multiples of the frequency of oscillation that represent the unfreezing of new degrees of freedom that take advantage of reactive forces (see introductory remarks). As the sessions progressed, these subcomponents were responsible for more and more of the relative motion associated with the coordination of the wrist-pendulum systems.[7] Subject 3 seems to have chosen a slightly different way to solve the coordination problem at hand, but he solved it nevertheless (e.g., Figures 6 and 7). Figure 13 shows the relative phase spectra of Subject 3 for Sessions 6-12. The early spectra (cf. Session 6) of this subject look very much like those of the other subjects (i.e., peaks at integer multiples; 70% of total power at peaks). Session 8-12 shows three peaks of minor magnitude (about 50% of total power), however, with the significant proportion of the power distributed over the base that the peaks rest on (about 25%) and a decreasing tail of power from about 5 to 12 on the frequency axis (the other 25%).

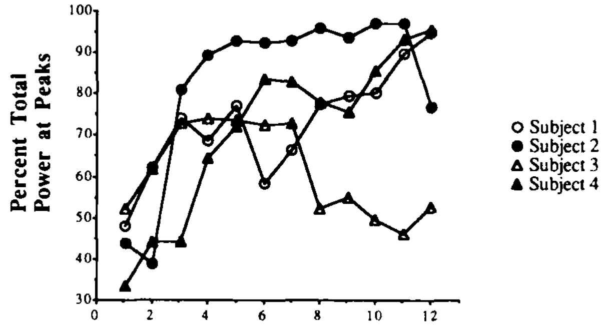

FIGURE 12. Summary of the progression in allocating power to integer peaks, shown as the percentage of total power contained in the integer peaks across all sessions for all subjects.

Further, the form of Subject 3's spectra appears to be very strongly a $1/f$-like distribution of power (i.e., the magnitude of power increases linearly with $1/f$), which is punctuated with the peaks at the low integer multiples. Regressions of log frequency on log power were significant for all 12 sessions. For Sessions 1-7, the regressions had a mean slope of -1.013, a mean intercept of -2.643, and a mean r^2 of .43. For Sessions 8-12, the regressions had a mean slope of -1.254, a mean intercept of -2.326, and a mean r^2 of .73. The slopes and r^2s of these two groups were significantly different by a t test: slopes, $t(4) = 5.22, p < .01$; r^2s, $t(4) = 3.47, p < .05$; the intercepts, however, are not different. The differences between the r^2s demonstrates that the nonpeak redistribution of power in Sessions 8-12 resulted in a large increase in $1/f$-like behavior. The difference in the slopes suggests that more power was concentrated at the lower frequencies of the spectrum for the later sessions. Although the regressions of log frequency on log power were significant for all sessions for the 3 other subjects as well, the r^2s, slopes and intercepts did not change between the first (Sessions 1-7) and second (Sessions 8-12) parts of the experiment (by t test $p > .05$), and the mean values of these measures were near the magnitudes of Sessions 1-7 of Subject 3 (Table 1). The qualitative changes in the distribution of the power (stated above) and the change in the r^2 associated with the $1/f$ distribution of power both suggest that Subject 3 was following the same strategy as the other subjects in Sessions 1-7, and switched to a predominantly $1/f$ distribution of power in Sessions 8-12.

A $1/f$ distribution of power is a generic strategy of complex systems to partition energy across the time scales in a manner such that the longer the time scale (the lower the frequency) the greater the power at that time scale (Shlesinger, 1987; West, 1988; West & Goldberger, 1987; West & Shlesinger, 1990). It has been proposed that these distributions are found in many biological organizations (spatial as well as temporal) because of their inherent stability. It appears that Subject 3 found a way to control the relative movements between the wrist-pendulum systems that utilizes this generic organizing scheme. His finding of this strategy as the way to optimize the dynamical control

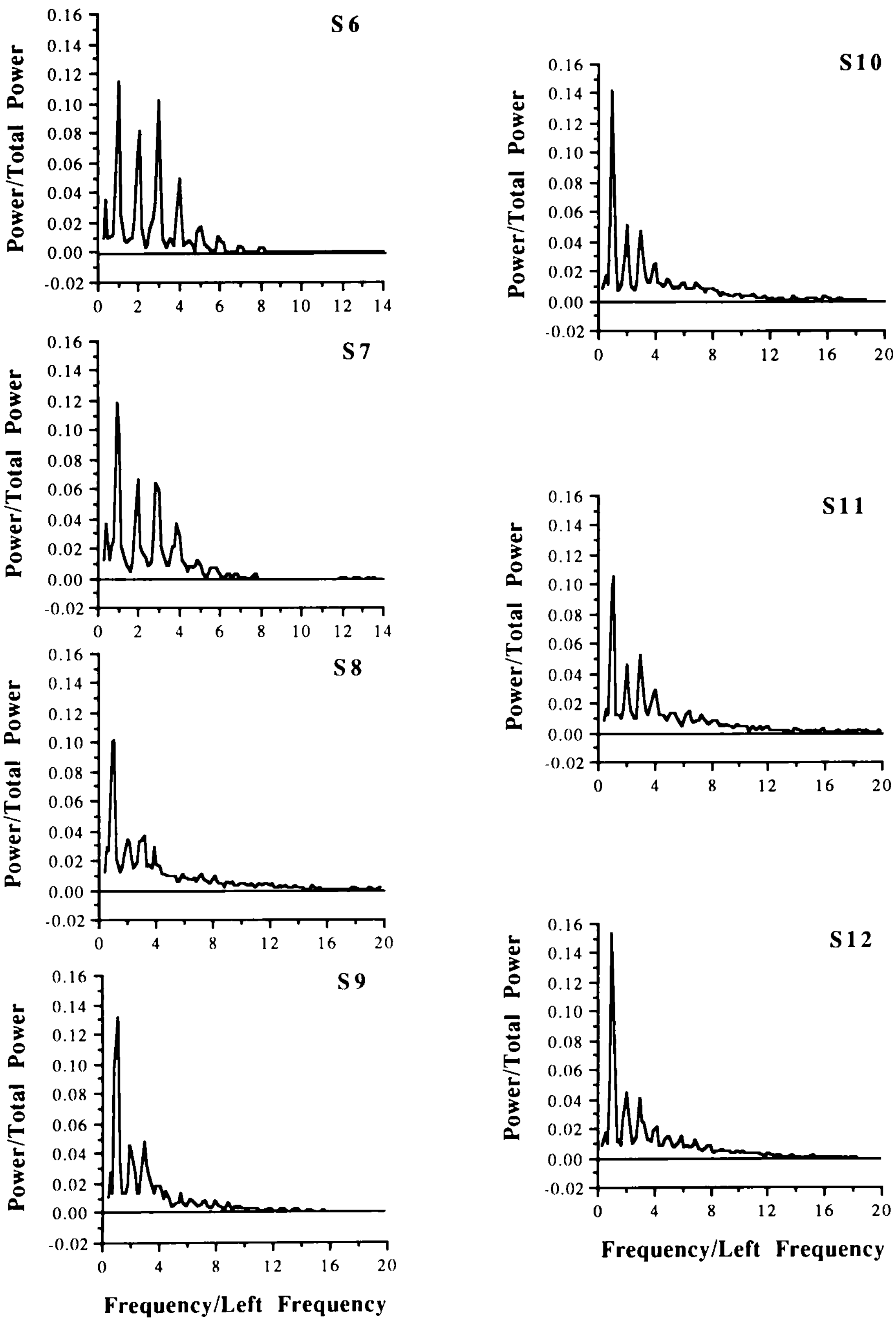

FIGURE 13. PVR spectra for Subject 3 across Sessions 6 to 12, showing the $1/f$-like strategy of allocating power to many noninteger frequencies.

TABLE 1
Mean Results of the Regressions of Log Frequency on Log Power for Subjects 1, 2, and 4 across the 12 Sessions.

Subject	r^2	Slope	Intercept
1	.37	−.89	2.87
2	.33	−.86	2.78
4	.42	−.94	2.78

structure governing the 2:1 frequency lock rather than the strategy of the other subjects may have been constrained by the frequency of oscillation that Subject 3 moved to in the later sessions. In the early sessions, when his oscillation frequencies were in the range of the other subjects, Subject 3's relative phase spectra were more like those of the other subjects. The evolution of his strategy may have been mandated by the frequencies of oscillation that he had chosen. This possibility is empirically verifiable by setting the frequency of oscillation of subjects in the range (right hand, 1 Hz; left hand, .5 Hz) chosen by Subject 3 in the later sessions. If the frequency of oscillation dictates the strategy chosen for constraining the relative motions of the wrist–pendulum systems in a 2:1 frequency lock, then one could speculate that scaling the frequency of oscillation from higher frequencies to lower frequencies might locate a critical frequency at which a bifurcation occurs from one control regime to another. That this is possible is suggested by past research that has found that changes in the frequency of oscillation cause a breakdown of relative phasing in 1:1 frequency locking and a switch from the alternate to symmetric modes of phasing (Haken et al., 1985; Kelso, 1984; Schmidt et al., 1990).

Conclusions

Previously, it has been proposed that dynamical principles constrain the control and coordination of skilled behaviors (Kugler & Turvey, 1987; Schöner & Kelso, 1988) and their learning (Newell et al., 1989; Schöner, 1989; Shaw & Alley, 1985). The basic assertion of this proposal is two-fold. First, as the learning of a skill proceeds, the dynamical regime that governs the control of the apposite movement system becomes more stable through increasing attunement to information about the task dynamics, specifically, its stability regime. Second, the intention of the subject to more closely approximate a stable organization can be considered to be the relaxation of an intentional attractor that has an equilibrium point at the goal level of skill.

An experiment was performed that explored the acquisition of a skill (2:1 frequency locking of rhythmic limb movements) from this perspective. Assuming that the time history of a single observable is all that is required to recover the dynamics of a system (Packard, Crutchfield, Farmer, & Shaw, 1980), and assuming a low-dimensional redescription of the coupled oscillator behavior, the ratio of the cycle frequencies of the two oscillators was used as an index of the dynamics of the action control structure through the course of the learning. Using this quantity, methods were employed to determine the shape of the potential field underlying the dynamic of the action control structure at each stage in the learning and to find the relaxation time of the intentional dynamics governing the learning process. The decrease in the fluctuations of the frequency-locking measure allows one to estimate that the concavity of the potential well governing the action control structure decreased as the sessions progressed (Figure 8). The decrease in the deviation of the frequency-locking measure from the goal state (2:1) permitted an estimate of the relaxation time of the intentional dynamic in the range of five to nine sessions (or 100 to 180 trials) for learning this behavior (Figure 6).

Analysis of the relative phasing of the two wrist–pendulum systems suggests that the increased stability of the coordination over the sessions was established as a consequence of certain qualitative changes in relative movements of the rhythmic units. The evolution of a stable topology of relative movements was found to occur in the analysis of the spectrum of PVR. For 3 of the subjects, the stable topology they progressed toward was characterized by peaks at integer multiples of the left hand frequency in the spectrum of PVR. The evolution of these peaks indicates that a more intricate patterning of relative movements, ones that possibly capitalize on the reactive forces of higher-frequency rhythmic subcomponents, produces the most stable frequency locking. The remaining subject found an alternative patterning of the relative movements of the rhythmic units that had a $1/f$ distribution of power in the PVR spectrum and was no less stable than the other subjects' strategy.

Of course, it must be stated that the present study obviously does not prove that a dynamical theory of coordination learning is correct. The phenomena observed in the study (e.g., task performance improving with practice, task variability lessening with practice, etc.) can be addressed from any number of perspectives on learning. What the present report does show, however, is that learning phenomena are amenable to dynamical descriptions and measures. The superlative aspect of a dynamical theory of learning is that it can account for the acquisition of a skilled behavior in a way that is continuous with the principles that guide the evolution of behavior in systems studied by the physical sciences. The dynamical approach to learning then holds out the promise of an integrated science of self-organization that can be used to understand changes in systems at all scales of nature and of different orders of complexity.

ACKNOWLEDGMENT

The research was supported by a grant from the National Science Foundation (BNS-8811510) awarded to M. T. Turvey. B. K. Shaw was supported by a National Science Foundation Predoctoral Fellowship. M. T. Turvey is also at Haskins Laboratories, New Haven. The authors wish to thank Charles Walter and an anonymous reviewer for their helpful comments.

NOTES

1. The latter constraints, which must be instantiated by neural processes, are the physiological, functional equivalents of the elastic potentials and forcing functions found in mechanical oscillators.

2. This view of the dynamical control structures that govern movements differs from the dynamic pattern generation approach. See Note 6.

3. Although potential wells as such represent point attractor dynamics, they have been used as alternative descriptions of the macroscopic patternings produced by the higher-order dynamics, for example, limit cycle and coupled limit cycle systems (Haken, 1977; Haken, Kelso, & Bunz, 1985; Kugler & Turvey, 1987).

4. It also is assumed here that an assembled dynamical control structure is structurally stable over the course of the action and, hence, that any parameterization of it (say, by information detected haptically or visually or both) does not change the topology of the dynamics of the system for the life of the control structure. Needless to say, the dynamics underlying complex movement patterns (e.g., serially ordered movements) require a complex definition that may be difficult to determine.

5. This normalization of the phase plane follows Beek and Beek (1988). We believe it leads to a clearer interpretation of phase plane deformations than the standard unit circle normalization. In the latter, displacement is scaled to peak displacement and velocity to peak velocity. Such a normalization may cause arbitrary deformations (and periodicities) by forcing confluence at the cycle extrema. The normalization of the phase plane performed here is one that is used in the study of the nonlinear differential equations (e.g., Jordan & Smith, 1977). In such a normalization, the velocity is scaled by angular frequency and the displacement is "centered" around the mean displacement, which reduces the units of the y-axis of the phase plane from radians per second to radians—the units of the displacement of the x-axis. For the trajectory of a harmonic oscillator, the normalization yields a perfect circle in the phase plane. Deviations from a perfect circle represent the presence of dynamics other than that of a harmonic oscillator. See Beek and Beek (1988) for further details on the use of this normalization in the interpretation of such topological deformations. Also, cycle averages calculated over the entire trial were used in this normalization because the dynamics are assumed to be stable at the time scale of the trial, and, hence, these statistics are the best estimates of the system's inherent dynamical properties, such as frequency.

6. Kelso and colleagues (Haken et al., 1985; Kelso, 1990; Schöner et al., 1986) have proposed a theory of the relative phasing of 1:1 frequency-locking rhythmic movements, which maintains that at 0° and 180° phase difference are point attractors that define the symmetric and alternate phase modes, respectively, and whose stability is manipulated by the frequency of oscillation. In this account, the relative phase difference is considered an order parameter, one that defines the topological ordering of the components, whereas the frequency of oscillation is considered a control parameter—a parameter whose magnitude controls the possibilities for ordering of the limbs. We embrace this dynamical account of relative phasing in a 1:1 frequency lock (Schmidt, Carello, & Turvey, 1990) but maintain that it is incomplete. What this account of relative phasing leaves out is the necessary perceptual tuning that must occur during the course of the coordination. Schöner and Kelso (1988) refer to the dynamics underlying the relative phasing of limbs as intrinsic, such that it functions without any parameterization or tuning by extrinsic environmental information. Instead, Schmidt et al. (1991) suggest for 1:1 frequency locking of rhythmic movements that perceptual processes are intrinsic to the organization of the control structure dynamic (point attractors at 0° and 180°) and provide evidence that there are discrete points within a cycle where perceptual information that tunes the relative phasing of limbs is picked up. The attractor dynamics in the present approach are spread across the actor and the environment rather than contained solely within the confines of the actor.

7. There are a number of relative position patterns that may satisfy the 2:1 frequency-locking task requirements. That is, the movements could synchronize in the middle of the cycle (peak velocity) or the cycle extremas. The increased harmonic nature of the spectra may indicate an increase in the ease by which the subject can move from one of these relative motion patterns to another.

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Submitted December 1, 1990
Revised June 1, 1991