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## Functional Stabilization of Unstable Fixed-Points

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How are the inherently unstable properties of a mechanical device actively controlled so as to achieve functionally stable behavior? In the language of dynamical systems this is the question of how one stabilizes an unstable fixed-point. Problems such as pole balancing in engineering (the basis of rocket flight), heart beat dynamics, and postural stability indicate the pervasive nature of inherently unstable systems. Learning may also be understood in terms of stabilizing hitherto unstable patterns of behavior (e.g., Zanone & Kelso, 1992). We propose that stabilizing an inherently unstable system requires perception-action strategies related to but somewhat different from those involved in controlling chaos. Whereas control of a chaotic system involves perturbing the dynamic onto one of an infinite number of pre-existing unstable but *periodic* orbits, stabilizing an unstable system requires continuous perturbation resulting in *nonperiodic* but functionally relevant behavior.

Consider balancing an inverted pendulum on the tip of the finger: If control stops, the pole drops. Mechanical models of pole-balancing use symmetrical bang-bang forces,  $f(x, \dot{x}, \theta, \dot{\theta})$ , whereas human performance results in asymmetric, nonuniform force application due to both functional asymmetries (Treffner & Turvey, in press) and those inherent in the task definition. Furthermore, human performance results in stabilization of the *top* of the pole relative to the bottom and hence is dependent on a subtle interaction of perception (visual and haptic) and action components of the task.

Since the displacement time series of the balanced pole appears to be a random walk, methods of stochastic time series analysis were employed. The Hurst exponent,  $H$ , as a measure of diffusion, quantifies the type of correlation between past and present points in a time series and captures the type of "constrained randomness." For Brownian motion, the mean square displacement (variance) in a 1-D random walk is related to the time step as

$$\langle \Delta x^2 \rangle = 2D\Delta t \quad (1)$$

where the diffusion coefficient,  $D$ , quantifies the extent of stochasticity. Mandelbrot generalized the above to include non-Brownian motion, that is, fractional Brownian motion:

$$\langle \Delta x^2 \rangle \sim \Delta t^{2H} \quad \text{where } 0 < H < 1 \quad (2)$$

The value of  $H$  is found from the slope of the linear region in the log-log plot of  $\langle \Delta x^2 \rangle$  versus  $\Delta t$ . Pure Brownian motion yields  $H = .5$  since no correlation exists between past and present points. With  $H > .5$ , the direction of past increments is preserved (positive correlation) and is called persistence. With  $H < .5$ , the direction of past increments is reversed (negative correlation) and is called antipersistence. Both persistence and antipersistence indicate that a system exhibits "memory" in that long-range correlations (structure or coupling) exist between past and present points in the time series. Over a range of different time steps,  $\Delta t$ , the same  $H$  may obtain and hence, the same type of stochastic dynamic relates all such separated points. However,  $H$  may switch from  $H > .5$  to  $H < .5$  at some critical point (CP) of  $\Delta t$  indicating the switch from persistent to antipersistent dynamics. The CP has been taken to indicate a switch in feedback strategy during human postural control from open to closed loop (Collins & De Luca, 1993, 1994).

We addressed the following issues: Is a CP in the Hurst exponent observed in human coordination tasks and does this help isolate a collective variable or order parameter that couples or relates action (of the hand at the bottom) and perception (of the top)? Can such relational variables specify upcoming events such that evasive and prospective control actions may occur before the perceived limits of controllability are exceeded?

## Method

Normal human adults attempted to balance an aluminum rod which could be held at its pivot (at the bottom) but was constrained to slide on a one-dimensional track of 180 cm in length. Rods of varying length permitted precise control of the natural frequency of the pole [ $T = 2\pi\sqrt{L/g}$ ]: 30, 45, 60, and 105 cm yielding frequencies of 0.91, 0.74, 0.64, and 0.49 Hz, respectively. Trials were 300 s in duration. Displacement time series were recorded at 100 Hz with an Optotrak infrared camera with LEDs placed on the bottom, the middle (center of gravity or CoG), and the top of the pole. Displacement data in the frontal plane provided a time series from which derivatives and Hurst exponents were obtained.

## Results

The displacement time series indicated that the top was displaced the least in comparison to the CoG and bottom of the pole. Hence, the top appeared to be the most stable point – as in a standard pendulum – and may imply that control of the top is crucial for achieving the goal of stability. The position-velocity phase plane (not shown) revealed that the dynamic was not simply a periodic limit cycle typical of bang-bang control. Instead, there were regions indicative of differential forms of stability. Tight cycles corresponded to the actors jiggling strategy for stabilization. In contrast, larger circuitous cycles were specific to a running strategy whereby the hand and the top of the pole were both displaced in the same direction until the impending end-of-track

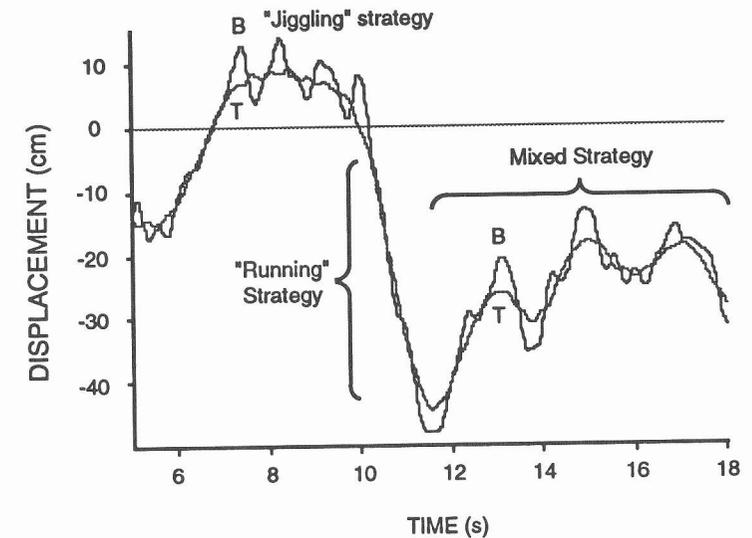


Figure 1. Displacement time series in the frontal plane for the top (T) and bottom (B) of the pole indicating the three major strategies employed. Note that the bottom is both continually displaced less than and encompassed by the top of the pole.

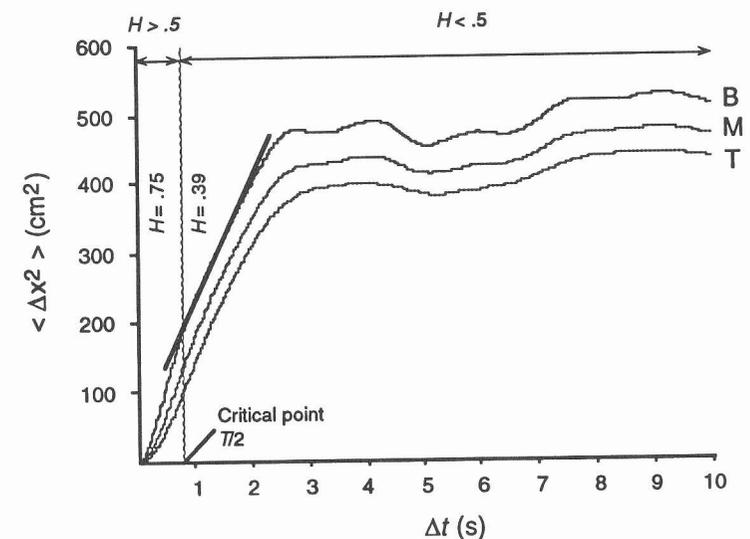


Figure 2. Hurst plot for one participant for  $0 < \Delta t < 10$  s with pole eigenperiod  $T = 1.55$  s. For the bottom,  $T$  determines the critical point where persistence,  $H > .5$ , switches to antipersistence,  $H < .5$ , and coincides with a value of  $\Delta t$  where the linear region of  $H$  leaves the curve. The value of  $H$  also changes along the pole. The additional periodicities imply that linear scaling may not apply at longer time-scales of  $\Delta t$ .

constraint was perceived, at which point another bout of jiggling recommenced. A third approach appeared to be a composite of both strategies (see Figure 1).

The peaks in the Fourier analysis revealed a rich spectrum of periodicities, but none had an obvious mechanical correlate. In contrast, the Hurst analysis revealed a CP of twice the natural frequency of the rod (Figure 2). The CP suggested a short, persistence time scale separated from a long, antipersistence time scale indicating fractal scaling in time. Unexpectedly, the antipersistent regime entailed additional periodicities which were before obscured by trial averaging (e.g., Collins & De Luca, 1993, 1994). A maximum in  $D$  is expected to occur at twice the natural frequency of a purely sinusoidal signal since the variance in Equation 1 will be maximal. Likewise, periodicities are to be expected in the  $H$  plot. Figure 2 also reveals a strong coupling between the top, CoG, and bottom for all  $\Delta t$ .

The stabilizing of unstable fixed-points is crucial for how biological systems maintain functionality under continually changing contexts. Mechanical properties influence the coordination dynamics and fractal time series analysis promises a way for determining how the dynamics is influenced by the mechanics. Our future use of visual occlusion and lateralization conditions will further probe the relation between perceiving and acting (e.g., Treffner & Turvey, in press; Treffner & Turvey, 1995)

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